VECTOR SPACE AXIOMS

Let V be an arbitrary nonempty set of objects on which two operations are defined: vector addition, $\oplus$, and scalar multiplication, $\odot$. If the following 10 axioms are satisfied for all $\vec{u},\vec{v},\vec{w}$ in V and all scalars *k* and *m* then V is called a Vector Space.

(1) The Closure Property of Vector Addition: $\vec{u},\vec{v} \in V \Rightarrow \vec{u} \oplus \vec{v} \in V$

(2) Commutative Property of Vector Addition: $\vec{u} \oplus \vec{v} = \vec{v} \oplus \vec{u}$

(3) Associative Property of Vector Addition: $\vec{u} \oplus (\vec{v} \oplus \vec{w}) = (\vec{u} \oplus \vec{v}) \oplus \vec{w}$

(4) Existence of Zero Vector: There exists a vector $"\vec{0}" \in V$ such that $\vec{u} \oplus \vec{0} = \vec{0} \oplus \vec{u} = \vec{u}$

(5) Existence of Additive Inverse: There exists a vector $"-\vec{u}" \in V$ such that $\vec{u} \oplus -\vec{u} = -\vec{u} \oplus \vec{u} = \vec{0}$

(6) The Closure Property of Scalar Multiplication: For any scalar k, and $\vec{u} \in V$, $k \odot \vec{u} \in V$.

(7) Distributive Prop.Vector Add. Over Scalar Mult. $k \odot (\vec{u} \oplus \vec{v}) = k \odot \vec{u} \oplus k \odot \vec{v}$

(8) Distributive Prop of Ordinary Add. Over Scalar Mult. $(k+m) \odot \vec{u} = k \odot \vec{u} \oplus m \odot \vec{u}$

(9) Associative Prop. Of Scalar Mult. $k \odot (m \odot \vec{u}) = (km) \odot \vec{u}$

(10) Unitary Property of Scalar Multiplication: $1 \odot \vec{u} = \vec{u}$
